CONTROL OF TRAVELLING WAVES IN REACTION-DIFFUSION BIOLOGICAL SYSTEMS

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Reaction-diffusion mechanisms are central in modeling a number of physiological systems such as those describing neural communication, cardiac rhythms or visual perception in the retina. One such model is that proposed by Hodgkin and Huxley and its subsequent simplified versions, among which the so-called FitzHugh-Nagumo (FHN) model (see [1]) is probably the best known. Slight variations of this model have been employed to represent traveling waves that induce the heartbeat or the formation of spirals and irregular fronts, responsible of arrhythmia or fibrillation phenomena of the heart. Dynamic analysis and control of diffusion-reaction systems, and in particular the FHN system, has been the subject of intensive research, especially in what refers to bifurcation analysis leading to moving fronts, spiral waves and pattern formation and their stabilization. In this regard, simple though efficient feed-back control schemes have been recently proposed to unpin or to command the evolution of meandering spiral waves (see for instance [2]).

On the other hand, theoretical work in the control of non-linear diffusion-reaction systems has been mostly focused on the stabilization of given stationary patterns by techniques which made use of the dissipative nature of this class of systems and results in non-linear control theory [3] to develop robust controllers which were able to stabilize arbitrary modes in diffusion-reaction systems.

In this work, we concentrate on the stabilization of inhomogeneous patterns in the FHN system when structural and parametric uncertainties are present. The FHN model presents a rich variety of behaviours due to the high dependence of the solution on the parameters of the system. Figures 1 and 2 illustrate this fact.

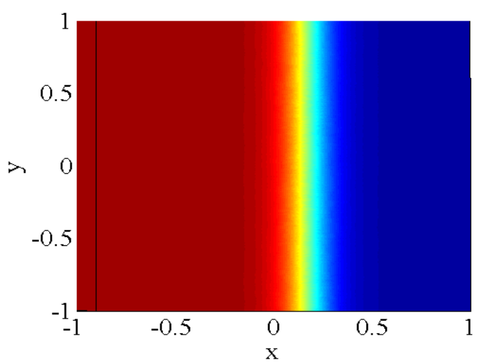

Figure1: Solution of the FHN model for a given set (*a*) of parameters

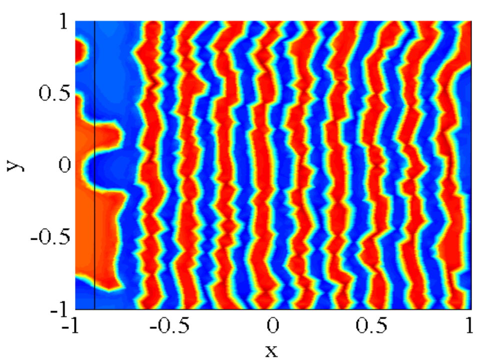

Figure 2: Solution of the FHN model for a given set (*b*) of parameters.

The solution with the form of squirming worms (Figure 2) is related with the fibrillation phenomena ([1],[4]) while the solution with the form of a plane front is related with the normal operation of the heart [4]. The control objective is to force a system exhibiting the behaviour in Figure 2 to evolve as that in Figure 1.To that purpose, we take advantage of the dissipative nature of diffusion-reaction systems and make use of results on model reduction of partial differential equations and non-linear control of finite dimensional systems to construct a class of robust feed-back non-linear controllers ensuring limit cycle stabilization. On a spatially distributed domain, this translates into front stabilization. In this way, we first demonstrate that a finite (and low) dimensional approximation of FHN system is always possible to be found and show that such a representation is in fact a good approximation of the original infinite dimensional system. On that basis a non-linear controller is built and its stabilizing and robustness properties demonstrated both at a theoretical level and through numerical simulations.

[1] Murray, J. Mathematical Biology I: An Introduction, 3rd Edition. Springer-Verlag, Berlin, 2002.
[2] Zykov, V., Engel, H. Feedback-mediated control of spiral waves. Physica D 199, (2004), 243-263.
[3] Alonso, A., Fernandez, C., Banga, J. Dissipative systems: from physics to robust nonlinear control. Int. J. Robust Nonlinear Control 14, (2004), 157:179.
[4] Centre of arrhythmia research. http://arrhythmia.hofstra.edu/ . Last visited on October the 6th, 2005.